LIGHTWEIGHT STRUCTURES in CIVIL ENGINEERING
CONTEMPORARY PROBLEMS
Monograph from Scientific Seminar
Organized by Polish Chapters of
International Association for Shell and Spatial Structures
Lodz University of Technology
Faculty of Civil Engineering, Architecture and Environmental Engineering
XXIV LSCE
Łódź, 7th of December 2018 (Friday)

COMPARATIVE ANALYSIS OF VARIOUS DESIGN SOLUTIONS OF OCTAHEDRON - BASED SPATIAL BAR STRUCTURES

D. Pilarska 1)
1) Ph.D., Eng, Faculty of Civil Engineering and Architecture, Opole University of Technology, POLAND,
d.pilarska@po.opole.pl

ABSTRACT: The regular polyhedra may constitute the basis of the spherical bar domes shaping. By the division of the original faces of the regular polyhedra into smaller, according to known methods of the division, we can obtain derivative polyhedra which can reflect grids of bar structures. The article concerns the spherical bar domes shaped from the regular octahedron. From the original regular octahedron, two families of derived structures were formed, by dividing its triangular faces into smaller ones. The detailed comparative analysis was carried out based on two designed spherical bar domes reflecting the first and the second method of the division. The analysis includes both topology and geometry of designed structures as well as static and strength parameters. Structures selected for comparative analysis have a diameter of 50 m and a comparable number of nodes and bars. The presented results can be the basis of the choice of spatial bar structures generated according to the various methods of the original octahedron face division. The diversity of geometric - topological shapes of designed bar domes can meet the requirements of modern and creative spatial structures.

Keywords: one-layered bar structure, structural topology, shaping of dome, comparative analysis.

1. INTRODUCTION

Among the building structures there are such structures, mainly spatial ones, for which is particularly important to link the three-dimensional work of complicated bars' systems. These include for example space trusses structures forming the spatial structures similar to domes, shells, etc. Such structures determine a particular challenge in topological and geometrical analysis, allowing them to design in the most rational way. Among mentioned structures, systems which the basis of shaping are regular polyhedra, deserve special attention. They are geodesic domes, whose precursor is Richard Buckminster Fuller. On the basis of icosahedron, as the base solid, he developed procedures for dividing the sphere into spherical triangles, thus creating the possibility of constructing lightweight, durable, self-supporting and economical covers (Ref. 1). Cost-effectiveness is related to the use of straight bars and, as a consequence, flat "faces" with little differentiation of edge length.

Fuller's concepts inspired architects and constructors to further design solutions for bar domes derived from the regular polyhedra, primarily from icosahedron and dodecahedron. The structures formed from the regularnoctahedron have been poorly developed so far. Therefore, in this study, methods of generating grids of bar domes based on a regular octahedron are characterized. Examples of designed covers are presented using two methods of dividing the original face of a regular octahedron. In addition to the detailed presentation of the geometric shaping of the designed systems, a static - strength analysis was carried out. The obtained results were subjected to a comparative analysis.

The designed bar systems are the original dome coverings. They are the determinant of the modern progress of the creative thought in the building design and construction.

2. METHODS OF THE DESIGNED OCTAHEDRON – BASED SPATIAL BAR STRUCTURES SHAPING

Bar domes are structures whose geometrical models are most often polyhedra grids inscribed in the sphere with a fixed radius. The basis for determining triangular spherical grids may be regular polyhedra. Due to appropriate transformations, we obtain derivative polyhedra with a greater number of faces. When it is assumed that the original polyhedron is the regular tetrahedron, octahedron or icosahedron, then after its transformations, polyhedra with a larger number of faces have all vertices on the concentric sphere and a relatively smallest number of groups of different edge lengths. The nodes of the structures correspond to the vertices of the selected polyhedron and the axes - to its edges (Ref. 2).

In Poland, Professor J. Fuliński, inter alia, dealt with the transformations of polyhedra (Ref. 3). He interpolated the flat, triangular face of the original polyhedron grid and obtained the division points thrown by the radial rays on the surface of the concentric sphere with the polyhedron. He developed three methods of the division of the spherical triangle.

The first method of the division of the original triangle leads to the division of each edge of the triangle into 2, 3, 4, …, n parts and to draw three families of lines parallel to each of the edge, as it is shown in Fig.1.

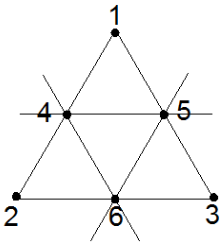

Fig. 1 The first method of face triangulation for a regular octahedron

In the second method, after dividing edges into n parts, three families of parallel lines are drawing to the lines of heights, each passing through the vertex and middle of the opposite edge. The presented method is shown in Fig 2.

The third method is also connected with the division of edges into *n* parts, where the further families of parallel lines are drawing to line passing through the vertex and one point laying on the opposite edge (not necessarily the first one, but also not the middle one), as it is shown in Fig 3 (Refs 3, 4).

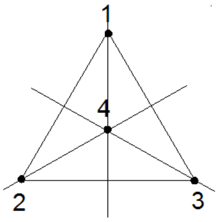

Fig. 2 The second method of face triangulation for a regular octahedron

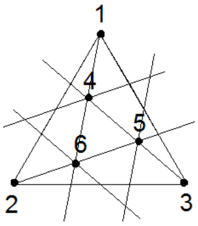

Fig. 3 The third method of face triangulation for a regular octahedron

3. THE TOPOLOGY OF THE DESIGNED OCTAHEDRON – BASED SPATIAL BAR STRUCTURES

Using two of the proposed by Professor J. Fuliński procedures of the division of the spherical triangle, the topology and geometry of the bar structures of the one-layered spherical domes were developed. The basis of their shaping is the regular octahedron. Further divisions of the original face of the octahedron cause the derivative polyhedra shaping with an increasing number of faces. Eight new structures were designed, the basis of which is the first method of dividing the original face of the regular octahedron and other eight new structures shaped according to the second division method. All bar systems are geodesic domes with a span of 50 m. For the comparative analysis of this article, two designed structures were used. They constitute the transformations of the regular octahedron. The number of nodes and bars are very close to each other. The first dome is the structure shaped from 4608-hedron, using the first described method of the division of the original face of the regular octahedron, the second one was generated on the basis of 4704-hedron, using the second method of the division. Figs 4 - 7 show both analyzed domes.

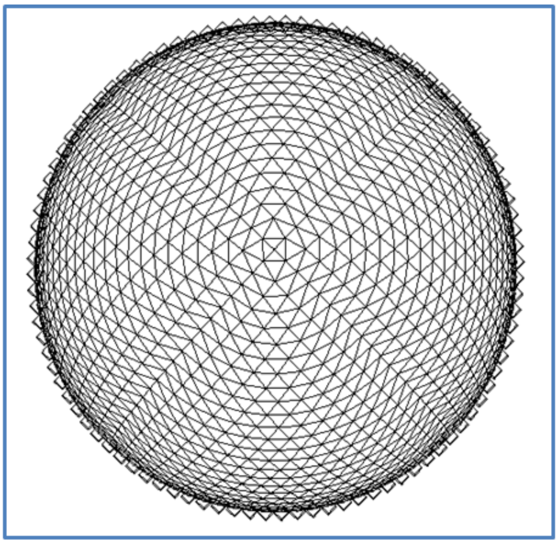

Fig. 4 The geodesic dome with the span of 50 m, created from 4608-hedron according to the first method of the division of the original face of the regular octahedron (structure no 1) – plan view

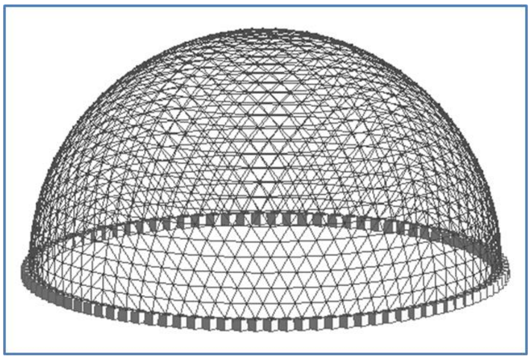

Fig. 5 The geodesic dome with the span of 50 m, created from 4608 hedron according to the first method of the division of the original face of the regular octahedron (structure no 1) – side view

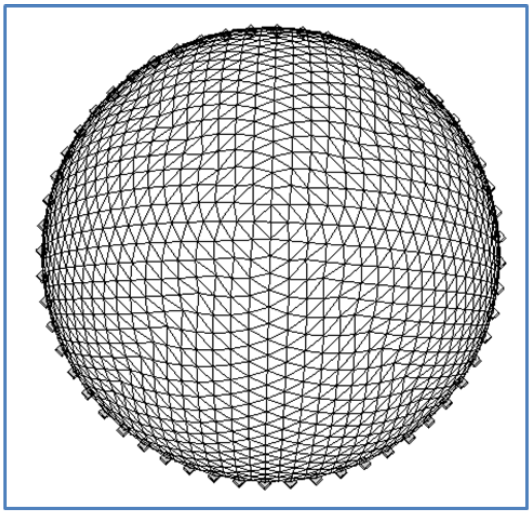

Fig. 6 The geodesic dome with the span of 50 m, created from 4704-hedron according to the first method of the division of the original face of the regular octahedron (structure no 1) – plan view

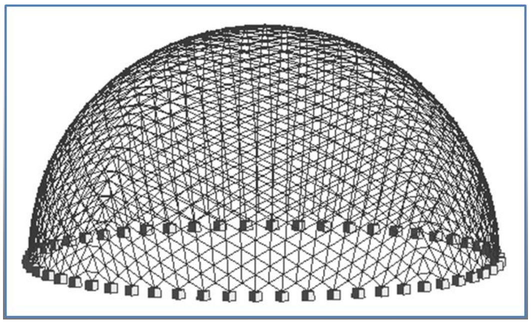

Fig. 7 The geodesic dome with the span of 50 m, created from 4704-hedron according to the first method of the division of the original face of the regular octahedron (structure no 1) – side view

4.GEOMETRY ANALYSIS OF THE DESIGNED OCTAHEDRON – BASED SPATIAL BAR STRUCTURES

The carried out geometrical comparative analysis includes the following parameters: number of nodes, bars and supports, number of groups of bars of different lengths, average number of elements in one group, minimum and maximum length of bar, total length of bars in a given dome. The variety of topology and geometry of the analyzed bar domes allowed to indicate this structure, whose parameters are more optimal.

The numbers of nodes and bars of both bar structures are very similar to each other, which was the basis for the choice of these two models to perform a comparative analysis. Bar dome, which is formed according to 4608-hedron, consists of 1201 nodes and 3504 bars. The second structure, derived from 4704-hedron, contains 1205 nodes and 3500 bars. These numbers are shown in Fig 8.

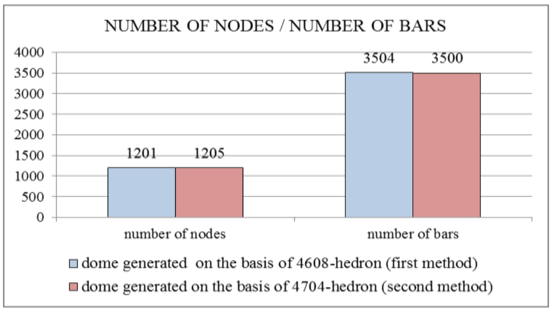

Fig. 8 The number of nodes and bars for the analyzed structures

The structure formed according to the first method has 96 supports, which is the result of dividing all edges of the original face of the regular octahedron into 24 equal parts. The system generated on the basis of the second method causes the division of each edge of the original face of the regular octahedron into 28 parts, but only half of them are led directly to the very edge. Finally, the second dome contains about 40% less supports than the first dome, i.e. 56. The comparison of the number of supports of the analyzed systems is presented in Fig 9.

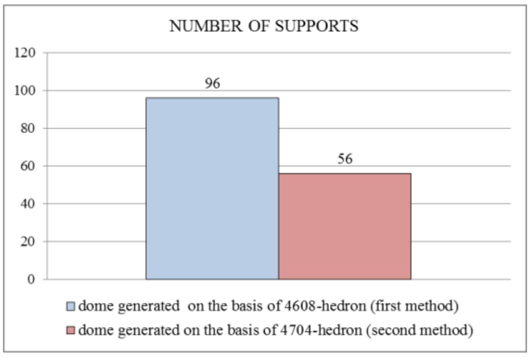

Fig. 9 The number of supports for the analyzed structures

When looking for a grid of elements for spherical surface, we aim to obtain the largest possible number of elements of equal length. Regular polyhedra fulfill these condition, and therefore they can constitute the basis for shaping the structures of spherical bar domes.

The first dome is described by 3504 bars, which are grouped into 101 groups of elements of different lengths. In each group there are on average 35 bars. The second dome is a structure containing 3500 bars, classified in 130 groups of bars of different lengths. In this case, there are about 27 elements in one group. The topology and geometry of the dome formed on the basis of the first method of the division of the original face of the regular octahedron is characterized by a smaller number of groups of bars of different lengths. The average number of bars of the same length occurring in one group is about 30% greater than in the dome shaped according to the second division method. The first structure is therefore more economical and easier to assemble. Figure 10 presents a graphical comparison of the number of bar groups of different lengths and average number of elements present in one group of bars of the same length.

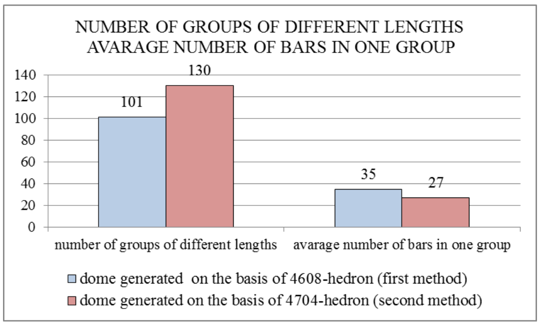

Fig. 10 The number of groups of different lengths and the average number of bars in one group for the analyzed structures

The next developed parameter is the minimum and maximum length of the bar, occurring in both analyzed domes. Approximation of the original face of the regular octahedron by means of two different division methods results in the separation of bars with different minimum and maximum lengths. The structure shaped according to the first division method contains bars with minimum lengths equal to 1.64 m, while the structure generated according to the second method has bars with a minimum length of 1.34 m, i.e. shorter by approx. 20%. The maximum lengths of elements present in both analyzed structures are similar to each other and differ only by approx. 2% (2,83 m for the first dome, 2,88 m for the second dome). The mentioned data are presented in Fig 11.

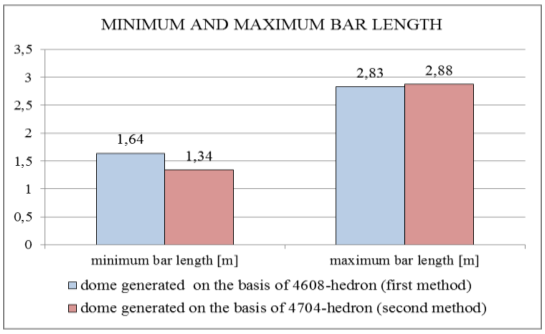

Fig. 11 Minimum and maximum bar length for the analyzed structures

Analyzing the lengths of bars of the designed two structures, the total length of all elements found in the discussed structures was also calculated. The bar dome generated on the basis of 4608-hedron has 3504 bars with a total length of 7101,56 m, while the structure shaped on the basis of 4704-hedron consists of 3500 bars with a total length of 7036,44 m. The lengths are close to each other, the difference is about 1%. This is due to the suitable choice of these two structures to perform a comparative analysis, the number of bars and nodes being very close to each other. The total length of all elements found in the designed domes is shown in Fig 12.

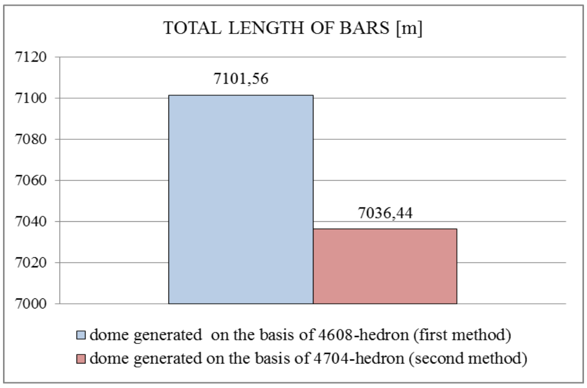

Fig. 12 Total length of bars for the analyzed structures

5. STATIC ANALYSIS OF THE DESIGNED OCTAHEDRON – BASED SPATIAL BAR STRUCTURES

5.1. Assumption

The static analysis of the developed eight bar domes was carried out in the Autodesk Robot Structural Analysis program. The following loads were taken into account: the fixed load, i.e. own weight of construction and weight of cover constituting glass panes with a weight of 0.6 kN / m2, as well as variable load, i.e. snow and wind for the first climate zone. From the presented interactions, the following load combinations were created: combination no 1 (KOMB1) containing fixed influences as well as leading variable influences of the wind and accompanying variable influences of the snow, combination no 2 (KOMB2) consisting of fixed influences as well as leading variable influences of the snow and accompanying variable influences of the wind, combination no 3 (KOMB3) including fixed influences as well as leading variable influences of the wind.

S235 steel with a yield strength 235 MPa was adopted. A round tube cross-section for all bars of the analyzed domes were assumed. Rigid supports were used, nodes were assumed as articulated.

5.2. Dimensioning

First, based on topological-geometric parameters, the analyzed domes were modelled. The bars of each structure were classified into 4 groups, taking into account a stress distribution in individual bars at the load of own weight. Models formed according to the 4608-hedron and the 4704-hedron, with grouped bars are shown in Figs 13 and 14.

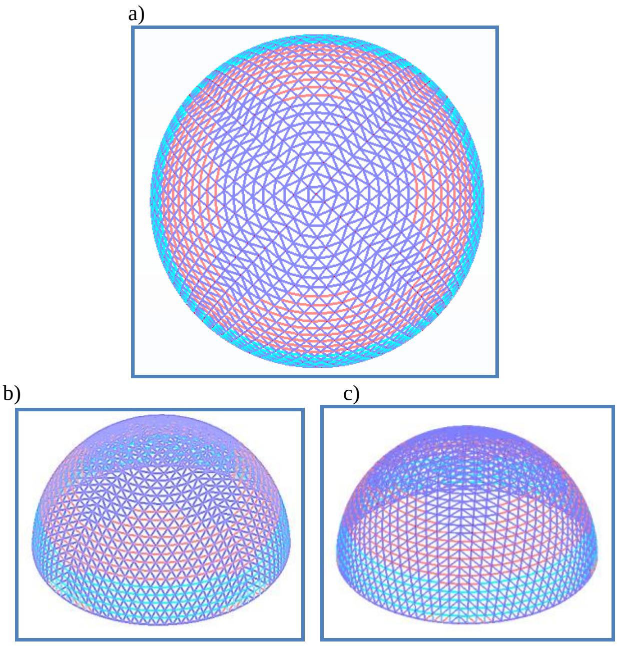

Fig. 13 Bars of dome formed of 4608-hedron, grouped into 4 groups: a) top view of the dome; b), c) side view of the dome

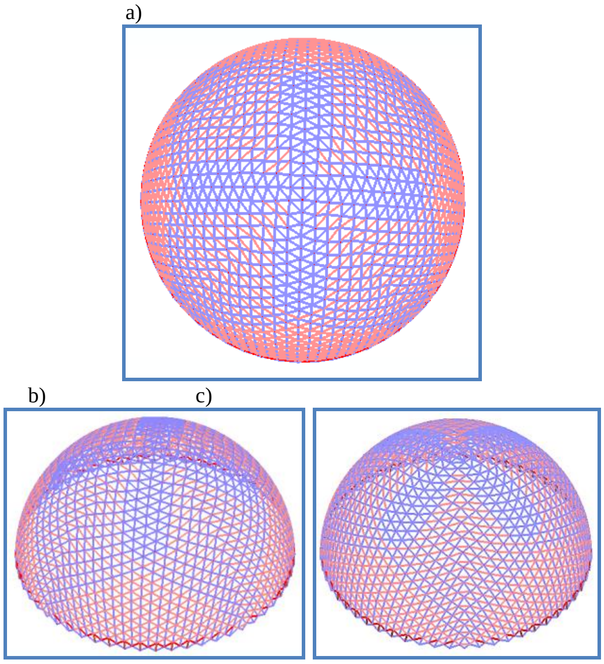

Fig. 14 Bars of dome formed of 4704-hedron, grouped into 4 groups: a) top view of the dome; b), c) side view of the dome

Elements in each group were assigned a cross section of the round tube, taking into account the usage of the most strained bars in the group at the level of 80-90%. List of individual groups of bars in a given dome together with the assigned cross section and the coefficient of bar tension is presented in Table 1.

Table 1. The division into groups of bars with assigned cross-sections and coefficients of bar tension in developed bar domes

	4608-hedron	**4704-hedron**
GROUP 1		
number of bars	48	136
cross-section	R 60,3x8,0	R 70,0x8,0
coefficient of bar tension	**0,86**	**0,90**
GROUP 2		
number of bars	2580	1164
cross-section	R 70,0x6,3	R 63,5x8,8
coefficient of bar tension	**0,90**	**0,88**
GROUP 3		
number of bars	472	1372
cross-section	R 42,4x5,0	R 57,0x8,0
coefficient of bar tension	**0,84**	**0,90**
GROUP 4		
number of bars	404	828
cross-section	R 48,3x6,3	R 44,5x6,3
coefficient of bar tension	**0,86**	**0,84**

Each cross section of the bars in a given group was analyzed in terms of its weight and finally the total weight of each designed system was obtained. The total weight of the second dome is about 9% larger than the first dome. In the structure generated from 4608-hedron, the group 2 is the heaviest. Its elements represent 74% of all the bars of the system. The assigned cross-section is characterized by high weight, which influenced the final weight of the entire structure. In the dome formed on the basis of 4704-hedron, the groups 2 and 3 have influence on the total weight. They include the largest number of bars (in group 2 - 33% of all dome elements, in group 3 - 39% of all dome components), which are assigned heavy cross-sections.

Table 2. The weight of bars in the individual group and the total weight of the dome formed of 4608-hedron

4608-hedron				
Group of bars	**Cross-section**	**Total length of bars [m]**	**Unit weight of bar [kg/m]**	**Total weight [kg]**
GROUP 1	R 60,3x8,0	489,92	10,30	5046,18
GROUP 2	R 70,0x6,3	4909,44	9,90	48603,46
GROUP 3	R 42,4x5,0	1294,88	4,61	5969,40
GROUP 4	R 48,3x6,3	407,32	6,53	2659,80
		7101,6		**62278,83**

Table 3. The weight of bars in the individual group and the total weight of the dome formed of 4704-hedron

4704-hedron				
Group of bars	**Cross-section**	**Total length of bars [m]**	**Unit weight of bar [kg/m]**	**Total weight [kg]**
GROUP 1	R 70,0 x 8,0	258,8	12,20	3157,36
GROUP 2	R 63,5 x 8,8	2292,04	11,90	27275,28
GROUP 3	R 57,0 x 8,0	2806,32	9,67	27137,11
GROUP 4	R 44,5 x 6,3	1679,28	5,94	9974,92
		7036,44		**67544,67**

5.3. Static analysis results

The designed bar domes were subjected to the detailed analysis of linear statics, including comparison of: extreme axial forces occurring in bars, maximum vertical and horizontal displacements of nodes and maximum and minimum values of stresses.

In both analyzed domes, the highest values of axial internal forces occur in the case of the combination of loads taking into account the impact of fixed influences as well as leading variable influences of the wind and accompanying variable influences of the snow (case of combination KOMB1). In the dome formed on the basis of the first method of the division of the original face of the regular octahedron, the extreme values of the internal compression and tensile forces of the bars are similar to each other and occur primarily in the support zone. Fig 15 presents a map of axial internal forces occurring in bars in the dome formed on the basis of 4608-hedron.

In the structure shaped according to the second method, the internal compressive forces are higher than the tensile ones. Their highest values also occur in the support zone. Most of the compressive forces are transmitted by bars lying on a straight perpendicular line to the base of the original face of the regular octahedron. Tensile forces occur in bars located perpendicular to the side edges of the original triangle. Fig 16 shows the map of axial internal forces occurring in the bars of the dome generated from 4704-hedron.

Table 4 presents the extreme values of axial internal forces occurring in the most disadvantaged combination of loads for both analyzed structures.

Table 4. Extreme values of tensile and compressive axial forces occurring in the most disadvantaged load combination (KOMB1) in developed bar domes

Dome	**Tensile axial force [kN]**	**Compressive axial force [kN]**
4608-hedron	-153,97	154,13
4704-hedron	-159,07	196,04

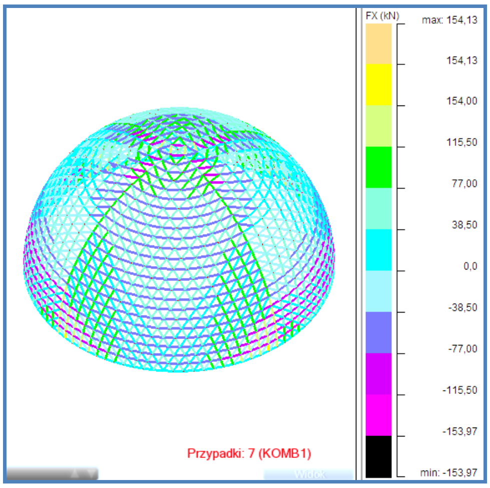

Fig. 15 A map of axial internal forces in the bar dome shaped on the basis of 4608-hedron, in the most disadvantaged load combination

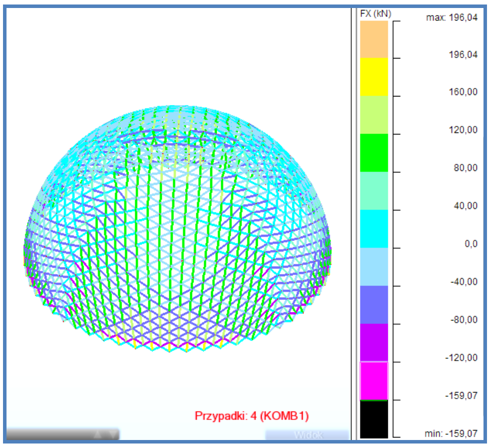

Fig. 16 A map of axial internal forces in the bar dome shaped on the basis of 4704-hedron, in the most disadvantaged load combination

Figure 17 shows the values of the occurred support reactions. In the dome created according to the second method of the division of the original face of the regular octahedron, both horizontal and vertical support reactions are much higher than in the first dome. The difference is approx. 140% for horizontal reactions and approx. 70% for vertical reactions. It is caused by a smaller number of supports at similar weight of the structure.

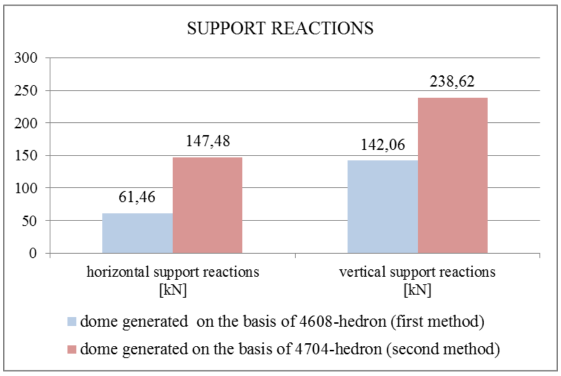

Fig. 17 Horizontal and vertical support reactions in the most disadvantaged load combination

Maximum horizontal displacements of nodes were noted taking into account fixed influences as well as leading variable influences of the wind and accompanying variable influences of the snow. Maximum vertical displacements of nodes occur in the case of the load combination taking into account the impact of fixed influences as well as leading variable influences of the snow and accompanying variable influences of the wind. The obtained results of the maximum displacements are presented in Fig 18. The distribution of vertical displacements in the most disadvantaged combination of loads is shown in Figs 19 and 20.

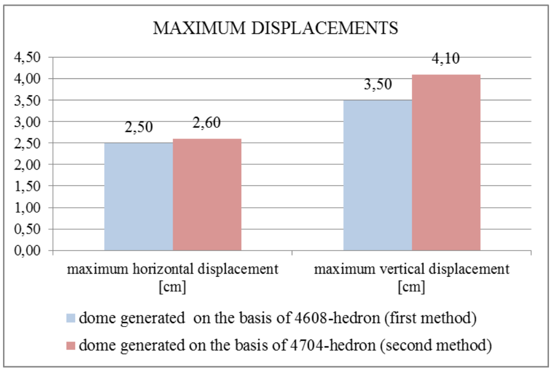

Fig. 18 Maximum values of vertical and horizontal displacements of nodes in the most disadvantaged load combination

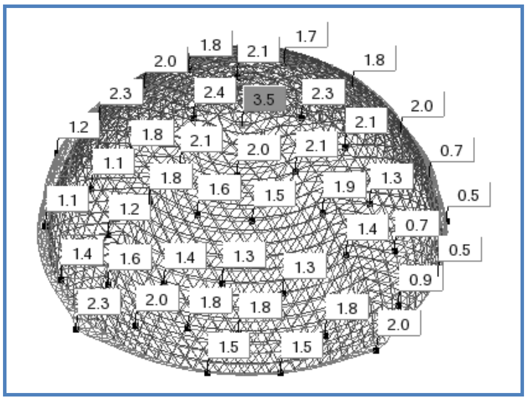

Fig. 19 Distribution of vertical displacements of nodes in the most disadvantaged load combination in the bar dome formed of 4608-hedron

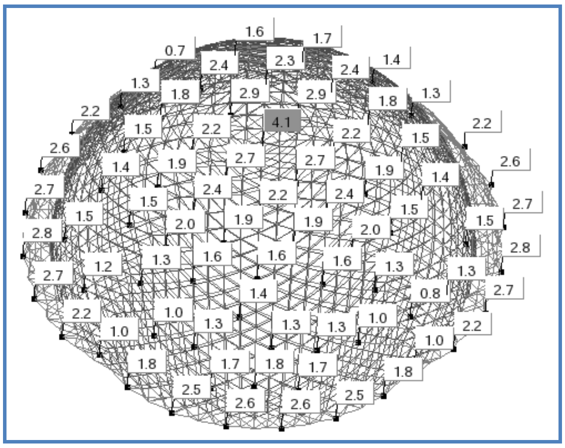

Fig. 20 Distribution of vertical displacements of nodes in the most disadvantaged load combination in the bar dome formed of 4704-hedron

Extreme values of normal stresses occur in the designed domes loaded by the fixed influences as well as leading variable influences of the wind and accompanying variable influences of the snow (case of combination KOMB1). These values are presented in Fig.21.

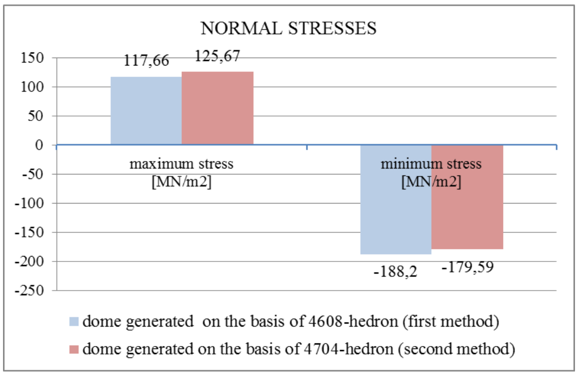

Fig. 21 Normal stresses in the most disadvantaged load combination

6. SUMMARY AND CONCLUSIONS

When looking for a grid of elements that is the basis for shaping spatial structures, including bar domes, we strive to obtain the largest possible number of elements with the same lengths. Such conditions are met by regular polyhedra. There are known studies on the use of dodecahedron and icosahedron to create bar domes. The other regular polyhedra, including the regular octahedron, have not been the subject of scientific considerations so far. Therefore, using the known methods of the division of the original triangular face of the regular octahedron, the author developed two families of spherical grids of bar domes. For the comparative analysis one bar structure was used, the basis of which is the first method of the division of the equilateral triangle (dome formed on the basis of 4608-hedron) and one bar structure generated according to the second method of the division of the equilateral triangle (dome formed on the basis of 4704-hedron). These structures have a very similar number of bars and nodes, which was the basis for the choice of these two bar systems.

The difference in the method of shaping of the analysed dome grids has an impact on the number of supports presents in the structures. The first dome has 96 supports, the second dome about 60% less, that is 56 supports.

An important parameter when choosing the appropriate structure topology is the number of groups of elements of the same length. In the system generated according to the first method of the division, the number is 101, while in the system obtained on the basis of the second division method there is 130. The dome shaped on the basis of 4608-hedron is therefore more advantageous in terms of economy and assembly.

The same diameter of the analyzed structures (50 m) as well as a comparable total number of bars, had an effect on the similar total length of all bars present in a given structure.

The bar systems were also subjected to static analysis. All bars were grouped into four groups, pipe sections were assigned to them, the total weight of the analyzed domes was given. The smaller number of supports due to the grid topology in the dome generated according to the second method of the division of the original face of the regular octahedron affects the value of support reactions. With a comparable weight of both structures, the difference is around 140% for horizontal reactions and about 70% for vertical reactions. The highest values of axial internal forces occur in the support zones of both considered domes.

The values of maximum horizontal and vertical displacements of nodes as well as the maximum and minimum stresses occurring in bars in the most disadvantaged combination of loads were compared.

The detailed comparative analysis, taking into account both the geometrical parameters as well as the stability of bar structures, may allow the designer to make a preliminary decision to select the appropriate topology and geometry of the bar structure type of the dome for design. Such systems can be used as covers of objects with large spans without the need for internal supports.

REFERENCES

1. B.R. Fuller: Geodesic Tent. United States Patent Office, patent 2, 914, 074, Nov. 24/1959.
2. J.Z. Mirski: Geneza i morfologia kopuł prętowych w aspekcie geometrycznego kształtowania form architektonicznych. Monografie, Studia, Rozprawy. Politechnika Świętokrzyska, Kielce 2003 r.
3. J. Fuliński: Geometria kratownic powierzchniowych. Prace Wrocławskiego Towarzystwa Naukowego. Seria B; nr 178/1973.
4. J.Z. Mirski: Siatki powstałe z przekształceń 8-ścianu foremnego. Zeszyty Naukowe Akademii Rolniczej we Wrocławiu. Melioracja XLI; nr 212/1992, pp.27-39.
5. Z. Kowal, J.Z. Mirski: Parametry geometryczne wybranej rodziny dwuwarstwowych kopuł prętowych. III Konferencja Naukowa: Konstrukcje Szkieletowe w Budownictwie Ogólnym i Przemysłowym ATR, Instytut Budownictwa Lądowego. Bydgoszcz – 24÷25.11.1982; cz.II, pp.255-267.
6. Z.S. Makowski: Raümliche Tragwerke aus Stahl. Verlag Stahleisen m.b.H. Düsseldorf 1963.
7. J.Z. Mirski: Numbers of edges and vertices in polyhedrons generated from regular polyhedrons. Silesian Technical University in Gliwice. Geometry and Engineering Graphics Centre. Proceedings of 4th Seminarium. Szczyrk 12-14.06.2003, pp.58-61.
8. J.B. Obrębski: Unidom-space bar system. Local Seminar of IASS Polish Charter; XII LSCE 2006; Warszawa 2006.
9. J. Rębielak: Struktury przestrzenne o dużych rozpiętościach. PNIAiU.PWr., Nr 27, Seria: Monografie Nr 15. Wyd. P.Wr. 1992.
10. T. Tarnai: Spherical Grids of Triangular Network. Acta Technica Academiae Hungaricae. Tomus 76, 3-4 1974.
11. D. Pilarska: Prętowe kopuły geodezyjne – propozycje przekryć dużych powierzchni. 62 Konferencja Naukowa Komitetu Inżynierii Lądowej i Wodnej PAN i Komitetu Nauki PZITB. Journal of Civil Engineering, Environment and Architecture, t.XXXIII, z. 63 (1/I/16), pp.447-454
12. D. Pilarska: Covers of large areas in the form of octahedron - based spatial bar structures. XXIII Lightweight Structures in Civil Engineering. Bydgoszcz 2017r., pp.41-46
13. D.Pilarska: Octahedron - based spatial bar structures – the form of large areas covers. 3RD Scientific Conference ENVIRONMENTAL CHALLENGES IN CIVIL ENGINEERING (ECCE). Opole, April 23rd-25th, 2018. MATEC Web Conf., Volume 174, 2018, Article Number 03007, DOI: https://doi.org/10.1051/matecconf/201817403007